Fraction Multiplication

Question:

How do you know that the answer to 2/3 x 18 is 12?

Possible answers to the question are:

1. If you divide 18 by 3 you get 6. Multiply 6 by 2 to get 12.
2. Imagine sharing 18 donuts equally among 3 people. Each person would get 6 donuts. Two people would get twice as much and that works out to be 12.
3. ‘Times’ is the same as ‘of’. 1/3 of 18 is 6. So 2/3 of 18 is double that, getting 12.
4. Imagine a string 18 metres long. Cut it into three equal parts. Each part would be 6 metres long. Two of those parts would be 12 metres long.

Response 1 is a procedural response. The method works for a fraction less than 1 times a whole number but it would be “interesting” to use for fraction multiplication questions such as 2/3 x 4/5. Also, the response tells you nothing about why the procedure works.

Response 2 involves thinking about the situation in a division way with the part of a set/group meaning of fraction. The explanation is appropriate but the method would be of little help for doing fraction multiplication questions such as 2/3 x 4/5.

Response 3 implies ‘of’ means multiplication. Key word approaches are always suspect. Consider this example. A grade 7 student quickly answered the following problem correctly by saying that the answer was 20%.

Four out <u>of</u> five coaches recommend heavy weight lifting as an important part of becoming an athlete. What percent of coaches do not recommend that?

The student's teacher was curious how the student obtained the answer of 20% so quickly. The teacher asked the student to explain how the answer was figured out. The student replied: ”I multiplied 4 x 5 because 'of' means multiply”.

Response 4 involves thinking about the situation in a division way with the part of a whole meaning of fraction. The explanation is appropriate but the method would be of little help for doing fraction multiplication questions such as 2/3 x 4/5.

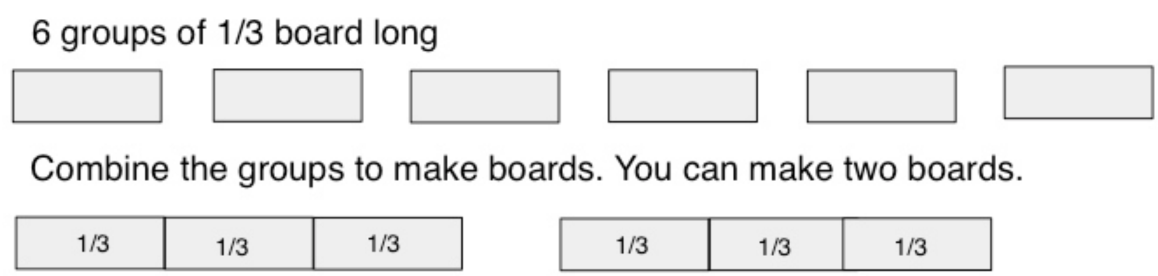
Models for teaching fraction multiplication

A variety of teaching models can be used to develop fraction multiplication. The measure, part of a whole, and part of a group/set meanings of fraction can be useful (see: Five meanings of fraction). However, these models are only useful for a whole number times a fraction or a fraction times a whole number. There is a model "out there" for a fraction times a fraction but it tends not to make sense to most students. [It is included at the end.]

Fraction bar (or circles) model (for fraction x whole)

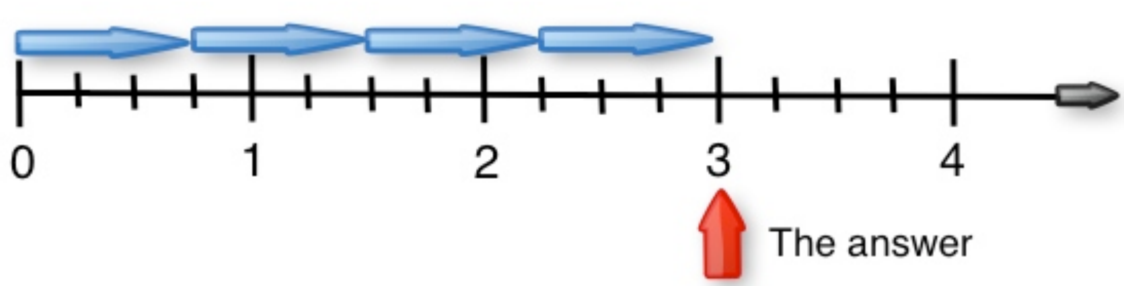
This model involves the part of a whole meaning of fraction. An example for 6 x 1/3 involving fraction bars follows.

Suppose you have 6 pieces of wood, each of which is 1/3 of a board long. How long would the 6 pieces be if they were combined? You can obtain an answer by doing 1/3 + 1/3 + 1/3 . . . but the question can also be expressed as: 6 x 1/3 = ? (This treats repeated addition in a multiplication way.) To obtain an answer to the multiplication, the model would involve making 6 groups of 1/3 and combining three at a time to make two sections each of which are 1 board long. Thus, 6 x 1/3 = 2.



Number line model (for whole x fraction)

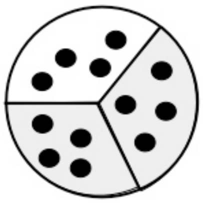
This model involves the measure meaning of fraction. An example for 4 x 3/4 follows.



The number line shows 4 steps (groups) each of 3/4 size. The answer (3) is shown at the end of the last step.

Partitioning model (for fraction x whole)

This model involves the part of a set/group meaning of fraction. An example for 2/3 x 12 follows.

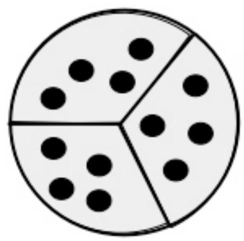


There are 8 marbles in the two-thirds part of the circle.

Suppose you have 12 marbles and distribute them equally (partition them) in a circle cut into thirds. There are 8 marbles in the shaded section (2/3 of the circle).

The issue with this model is that, for example, in 2/3 x 12, the '2/3' cannot refer to the number of groups (groups must be whole numbers) and thus it is not necessarily obvious to students why the diagram shows multiplication.

One way around this is to show a diagram of 3/3 x 12 (same as 1 x 12). This diagram would involve a circle cut into three parts, with 4 marbles in each third and all three thirds shaded. Discussion would be needed about why the picture shows multiplication (same as 1 x 12 = 12).



There are 12 marbles in the three-thirds parts of the circle.

Intersection model (for fraction x fraction)

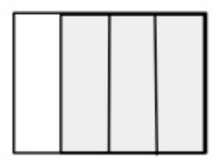
- This model involves the part of a whole meaning of fraction. A rectangle (fraction bar) must be used. It is the model that students may not find sensible. An example for 2/3 x 3/4 is shown here.

Cutting 2/3 at right angles to 3/4 cuts the rectangle into 12 equal parts. Of those 12 parts, 6 of them are double shaded. In other words, the intersection of 2/3 and 3/4 is the answer to the multiplication.

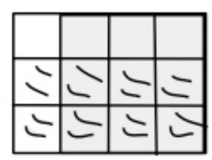
Thus, 2/3 x 3/4 = 6/12.

This model is presented here for your information. It is not used in the 3 stages plan for developing fraction multiplication.

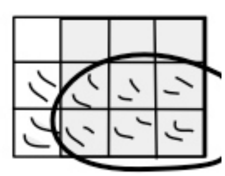
Step 1: Draw 3/4



Step 2:
Draw 2/3 at right angles to 3/4



The answer is the intersection of 3/4 with 2/3 (the double shadings)



Refer to: [Grade 8 Fraction multiplication (8.N.6)] **if more help is needed.**